

SUPPLEMENT TO SECTION 8.1

Portfolios with maximal Sharpe ratio

We shall here give the following result.

Proposition S8.1 *Consider the portfolio that has the stock weights \mathbf{v} , and the remainder in cash.*

(a) *Show that the Sharpe ratio of the portfolio equals*

$$\frac{\mathbf{v} \cdot (\boldsymbol{\mu} - r_f \mathbf{1})}{\sqrt{\mathbf{v} \cdot Q \mathbf{v}}}$$

regardless the amount of cash.

(b) *Show that \mathbf{v} maximizes the Sharpe ratio if and only if $\mathbf{v} = c\mathbf{v}_{max}$ for some $c > 0$, and that the maximal Sharpe ratio equals σ_{max} .*

Proof.

(a) The Sharpe ratio equals

$$\frac{(1 - \omega)r_f + \mathbf{v} \cdot \boldsymbol{\mu} - r_f}{\sqrt{\mathbf{v} \cdot Q \mathbf{v}}} = \frac{\mathbf{v} \cdot (\boldsymbol{\mu} - r_f \mathbf{1})}{\sqrt{\mathbf{v} \cdot Q \mathbf{v}}}$$

because $\mathbf{v} \cdot \mathbf{1} = \omega$.

(b) First maximize the Sharpe ratio under the constraint that the portfolio volatility equals σ , and then maximize over σ . The first maximum is according to (6.34) attained for $\mathbf{v} = c\mathbf{v}_{max}$ with $c = \sigma/\sigma_{max}$. It follows from (6.26) and (6.27) that the maximal Sharpe ratio equals σ_{max} , and hence is independent of σ . \square