SUPPLEMENT TO SECTION 4.2

Expected return and drift

Replace Exercise 4.12 and the text below it on page 68 by the following:

Note that the moment assumption does not imply that the first two moments of the returns exist. We shall therefore assume that this is the case.

The next proposition holds without this extra assumption.

Proposition S4.1 *The moment assumption implies that*

$$G(t, t + \partial t) = O(\sqrt{\partial t})$$

and

$$R(t, t + \partial t) = G(t, t + \partial t) + \frac{1}{2} G(t, t + \partial t)^2 + O(\sqrt{\partial t}^3)$$

in probability, as $\partial t \to 0$.

Proof. By Chebychev's inequality

$$P(|G(t,t+\partial t)-\nu\partial t|>A\sqrt{\partial t})\leq\frac{\mathrm{Var}(G(t,t+\partial t))}{A^2\partial t}=\frac{\sigma^2}{A^2}$$

for any $A>0$. The first statement follows from this.

It follows from Taylor's formula that the error in the second approximation equals $O(|G(t,t+\partial t)|^3)$. $\square$

The identities of the proposition suggest that

$$ER(t,t+\partial t)=\nu\partial t+\frac{1}{2}\sigma^2\partial t+o(\partial t)\text{ and }\mathrm{Var}(R(t,t+\partial t))=\sigma^2\partial t+o(\partial t).$$

Assume that these identities hold, and define

$$\mu=\lim_{\partial t\to 0}\frac{ER(t,t+\partial t)}{\partial t}\text{ and }\sigma_r^2=\lim_{\partial t\to 0}\frac{\mathrm{Var}(R(t,t+\partial t))}{\partial t}.$$

Then

$$\mu=\nu+\frac{1}{2}\sigma^2\text{ and }\sigma_r=\sigma.$$

We shall now also give an empirical verification of the latter identities. Let

$$r_k=r_k(\partial t)=R((k-1)\partial t,k\partial t),\quad k=1,...,n,$$

where $\partial t=1/250=$ one day.