SUPPLEMENT TO SECTIONS 2.2 AND 2.3

In sections 2.2 and 2.3 we show that certain inequalities hold on a market without arbitrage opportunities. One way to do this is to assume that the opposite inequality holds, and then construct an arbitrage by buying what is cheap and sell what is expensive.

Example In order to prove the Put-Call Parity we first assume that

$$e^{-rT}K + C_0 > S_0 + P_0$$

and show that this leads to arbitrage (and then assume that the opposite inequality holds and show that also this leads to arbitrage). Here we shall buy the right hand side, and sell the left side. □

It is clear what is meant by buying assets such that stocks and options, but what is meant by buying cash? And what is meant by selling an asset you do not own? To sell means in this context that you recieve money, and to buy means that you give away money.

Therefore:

Buy cash: Lend.

Sell cash: Borrow.

Sell an asset you do not own: Take a short position in the asset. This means that you borrow the asset and sell it at once (and buy it back later).

Sell an option: Write an option.

The latter is more straightforward than to borrow the option.

Example (Continued) Buy a stock and a put. Borrow $e^{-rT}K$ EUR and write a call. The value of the portfolio at time 0 then equals

$$F_0 = e^{-rT}K + C_0 - S_0 - P_0 > 0.$$

This amount is kept on an account. The value of the portfolio at maturity, T, is therefore

$$F_T = e^{rT}F_0 + S_T + P_T - e^{rT}(e^{-rT}K) - C_T = e^{rT}F_0 > 0.$$

We have thus constructed the cash stream $(F_0, e^{rT}F_0)$ both components of which are strictly positive. Arbitrage. $\square$