Contents with comments

MATHEMATICAL ASSET MANAGEMENT by Thomas Höglund

1 INTEREST RATE 1

Comment: One essential thing in this chapter is to give an understanding of the connection between present value and arbitrage, and how to construct arbitrage.

2 FURTHER FINANCIAL INSTRUMENTS 23

Comment: In this chapter we introduce a few financial instruments that besides bonds can serve as building stones for portfolios. Identities and inequalities for the prices of forwards and options are derived under the assumption that the assets are traded on an arbitrage-free market.

3 TRADING STRATEGIES 37

Comment: An interesting feature of this chapter is perhaps what we do not do: We do not start by making assumptions about the stochastic behavior of stock prices.

We show how to manage risks by trading under the sole assumption that stock prices are reasonably continuous. We also show by an example that the method works in practice even for a stock which is quite wild, and therefore hardly continuous.

If one assumes that stock prices really are continuous, and that stocks can be traded in arbitrarily small amounts one gets a neat asymptotic approximation by trading infinitely often. This result will be used later in Chapter 5 to derive the Black-Scholes formula.

4 STOCHASTIC PROPERTIES OF STOCK PRICES 53

Comment: The most frequently used stochastic model for stock prices is geometric Brownian motion. This model implies that the logarithmic returns are normally distributed. It is wellknown that this does not hold in reality for logarithmic returns over short time intervals. This will cause problems. It is, for example, not immediately clear how to define volatility.

We do not propose a parametric model that is more realistic than geometric Brownian motion. Instead we verify empirically that stock prices have certain properties that will imply that the volatility is well-defined, and also that some identities implied by the Gaussian assumption hold in reality. Some of these identities, but not all; the variance of the volatility estimator is unfortunately not as small as the Gaussian assumption implies.

For the remainder of the book, it is of utmost importance to be able to estimate the volatility in particular, and covariances of returns in general. They are needed to calculate the weights for the different trading strategies in the following chapters. We show that it is possible to this, but that it is not possible to estimate the expected return of a single stock with sufficient precision.

In this chapter we also make clear the difference between expected return and expected growth (drift). High drift is desirable.

It is easy to construct examples where the expected return takes any value for a given drift, but it is the drift that determines the long-term development of an asset. This insight will have consequences for Chapter 6. Here we also introduce the "*Five Stocks*", the development of five stocks and an index during a four year period. The data will be used throughout the book to illustrate different methods and portfolios.

5 TRADING STRATEGIES WITH CLOCK TIME HORIZON 75

Comment: The asymptotic result in Chapter 3 concerns trading strategies where the number of trades is given in advance. In this chapter the trading does not stop after a certain number of trades, but at a certain time. Here we need (for the first time) a stochastic model to relate this number to this time. This is the way we derive the Black-Scholes formula.

We also derive this formula in a more conventional way via the Black-Scholes equation. The reason for this is that this technique can be used to handle trading strategies for several assets (portfolios), and not just cash and a stock.

Again I think it is worth mentioning what we do not do: We do not use Ito calculus. Ito calculus can be used in Section 5.3, but I prefer a direct derivation because it is shorter if one wants to give a rigorous treatment, but also because I believe that students have a tendency to focus on the calculus and forget about the risk management.

6 DIVERSIFICATION 99

Comment:

Here we derive optimal portfolios under various constraints. This is done directly without making any reference to the efficient frontier. It turns out that all optimal portfolios are linear combinations of three portfolios: cash, the minimum variance portfolio and a certain auxiliary stock portfolio.

A trading strategy for several assets as considered in Sectioon 5.4 can be specified by the weights of the assets. These weights may depend both on time and the assets prices. The optimal portfolios in Chapter 6 can be considered as trading strategies with fixed weights (that do not depend on time or the asset prices).

In Section 6.3 we show that the value of a frequently rebalanced portfolio equals the geometric mean of the asset values multiplied by a certain factor. We also verify that this holds in reality.

Comment: In order to construct the growth portfolios of the preceding chapter, we must be able to estimate the differences between the expected returns of the stocks and the short rate. The Capital Asset Pricing Model in this chapter reduces this problem to the problem of estimating the difference between the return of the market and the short rate. As a consequence, all portfolios are known if we can predict the return of the market, and the portfolio that has maximal drift for a given volatility is completely known if we can predict only the sign of this difference.

We also have a case study to illustrate how the portfolio theory given in this and the preceding chapter works in practice by formulating different scenarios, and constructing the corresponding optimal portfolios.

Comment: The probability that one asset will perform better than another is a simple function of a certain ratio. In the case when one of the assets is the short rate this ratio is related (but not identical) to the Sharpe ratio.

The downside risk is not as uncomplicated as a risk measure as it seems at first sight.

Comment: Simplified covariation structure has three advantages: 1. It gives more stable estimates. 2. It gives simple and lucid explicit expressions for portfolio weights and other quantities, and this will give a better understanding of what is important. 3. There is a simple solution to the problem of finding optimal portfolio weights under the constraint that these must not be negative.

Comment: In order to avoid burdening this book with too many details, there are many exercises scattered through the text. These are a vital part of the book. Solutions to the theoretical exercises, along with answers to the others, are given here.