STOCKHOLM UNIVERSITY
Dept of Mathematics
Div of Mathematical Statistics
Advanced Finance Mathematics
Spring 2009

COMPUTER ASSIGNMENT 3

CAPM, growth, and rebalancing.

The data for this assignment is the same as for Assignment 2.

Task 1 Is the interest rate almost constant?

In the book we have considered cash as being a non-volatile asset (constant interest rate), but it isn't. You shall here check if the assumption we have made is acceptable by calculating the volatilities of the interest rate and the index. Choose a few fairly long sub-periods. Also calculate the correlation coefficient between the growths of cash and the index for these periods. The volatilities shall be expressed on a yearly basis.

Task 2 Growth portfolios.

According to CAPM the Tangency (or Markowitz) portfolio coincide with the market portfolio, which in this case is the omxs30 index. Accept this as a truth.

A consequence of this is that the maximal drift portfolio (without constraints), and the portfolio that has maximal drift for a given volatility, σ, consist of cash and the index only. The proportions are functions of $\Delta = \mu_M - r_f$, and σ_M (and σ in the latter case). Here μ_M and σ_M are the expected return and volatility of the market and r_f the interest rate. Both Δ and σ_M vary with time. In order to avoid the estimation problem we shall try to remove the influence of these parameters as far as we can.

Choose a number q, $0 < q < 1$. Assume that $\Delta > 0$, and determine the weights of the portfolio that has maximal drift under the constraint $\sigma = q\sigma_M$.

Neglecting transaction costs, the value of the continuously rebalanced portfolio then is of the form (6.47), where L is as in (6.48). (It is **not** as on the line above (5.36).) You don't have to rebalance this portfolio, but just use the formula (6.47) to calculate its value. When estimating L you shall use the actual volatility during the period you consider, because this is what you should obtain if you had rebalanced.

You shall compare this active portfolio with the passive portfolio (Buy and hold). That is, the portfolio that has the right weights at start, but which is never rebalanced.

An advantage with the passive portfolio is that if the market rises the weight of the good asset (the index) will increase, and if the market falls the weight of the bad asset (the index) will decrease. An advantage with the active portfolio is that it will gain on the volatility via the factor $\exp(Lt)$.

One possibility is therefore that the passive portfolio will be the best performer during periods that have a pronounced trend, and that the active will perform better after a trend shift. The latter portfolio may also perform better in a volatile market without a trend. Both portfolios can be expected to perform poorly on a bear market because then the assumption $\Delta > 0$ is violated.

But this is just speculations. How is it? Compare the developments of the two portfolios. Choose a few relevant periods, and illustrate with plots.

Task 3 The portfolio volatility.

We have tried to construct a portfolio that has good growth under a constraint on the volatility, provided that $\Delta > 0$. You shall here check how well we have succeeded with the constraint.

The volatilities will vary with time. Choose an appropriate integer n, and calculate the actual volatilities of the active, the passive, and the market portfolios under the periods $[i, i+n]$, $i = 1, 2, \ldots$. Compare the volatilities of the two first portfolios with the volatility of the market multiplied by q. A plot may help here.

Task 4 Conclusions and comments.